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COMMENTS ON "FREE VIBRATION ANALYSIS OF CYLINDRICAL TANKS PARTIALLY FILLED WITH LIQUID"

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I am writing this comment concerning the paper of Professors Gonçalves and Ramos [1] for two reasons: (i) to congratulate the authors for their very important and interesting paper that, in the opinion of the present writer, can be considered the reference for those interested on this topic; (ii) to discuss a secondary but interesting aspect of their study.

In the case of axisymmetric vibrations ($k = 0$) the form of their equation (27) should be:

$$\Phi = \sum_{m=1}^{NMF} A_m \cos(\zeta_m \xi) \mathrm{I}_0(\zeta_m r) \sin(\omega t) + \left[B_1 + \sum_{n=2}^{NMS} B_n \cosh(\alpha_n \xi) \mathrm{J}_0(\alpha_n r) \right] \omega \sin(\omega t), \quad (1)$$

where also the coefficient B_1 must be included in the expansion of the velocity potential of the liquid associated to the sloshing. Obviously expression (1) is obtained also by equation (27) of Gonçalves and Ramos [1], if one considers that the first root of equation (28) is zero for $k = 0$. The coefficient B_1 satisfies the Laplace equation and the boundary condition at the rigid bottom. Therefore it will be present in the sloshing equation (33). In the case of sloshing in a rigid tank, B_1 is neglected because it is zero when the sloshing condition is applied. However in the case of a flexible shell the sloshing equation is verified by a non-zero value of B_1.

This coefficient has also a physical meaning: it appears in axisymmetric modes as a consequence of the movement of the mean free liquid's surface to guarantee the preservation of the volume of the liquid. On the contrary, asymmetric modes ($k \geqslant 1$) do not present this coefficient in the velocity potential, and the physical reason is that, for these modes, the preservation of the liquid's volume is obtained without movement of the mean free surface.

In order to verify if the coefficient B_1 affects natural frequencies of axisymmetric modes, some numerical results are carried out including or neglecting this coefficient in the study. The case examined is that of a simply supported circular cylindrical shell closed by a rigid bottom and partially filled with water having a mass density $\rho_F = 1000$ kg/m^3; the sloshing condition is imposed on the free surface of the liquid. The following dimensions and material properties of the shell are assumed in order to compare with the results of Kondo [2], obtained by using the Goldenveizer theory of shells, with those of Amabili [3], obtained by neglecting the free surface waves, and with those approximate ones of Gupta and Hutchinson [4]: radius $R = 25$ m, shell height $L = 30$ m, water depth $H = 21{\cdot}6$ m, shell thickness $h = 0{\cdot}03$ m. The shell is considered to be made of steel having the following material properties: Young's modulus $E = 206$ GPa, mass density $\rho_C = 7850$ kg m^{-3} and Poisson's ratio $\nu = 0{\cdot}3$. The Flügge theory of shells is applied in this case. The circular frequencies of axisymmetric ($k = 0$) bulging modes of this shell are reported in Table 1, and show a small difference in the case when the coefficient B_1 is neglected with respect to the case when B_1 is retained. However, this coefficient plays an important role. In fact, if it is neglected, the results are very close to those obtained when neglecting the effect of

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TABLE 1

Circular frequencies (rad/s) *of bulging axisymmetric modes* ($k = 0$) *of the first studied shell*

Mode	Retaining B_1	Neglecting B_1	Amabili [3] (neglecting free surface waves)	Kondo [2]	Gupta and Hutchinson [4]
1	22·244	22·229	22·228	22·096	22·349
2	44·010	44·007	44·004	43·762	44·170
3	57·191	57·190	57·187	56·829	58·244
4	67·291	67·291	67·288	66·888	69·513
5	75·850	75·849	75·847	75·347	79·189

TABLE 2

Circular frequencies (*rad*/*s*) *of bulging axisymmetric modes* (k = 0) *of the second studied shell*

Mode	Retaining B_1	Neglecting B_1
1	1438·5	1440·8
2	5462·1	5463·6
3	8700·6	8693·4
4	10560	10566
5	12079	12077

the free surface waves (i.e., imposing $\Phi = 0$ at $\xi = H/L$). For sloshing modes, not significantly different natural frequencies are found for the same shell.

Then, a second simply supported shell is studied in order to verify that the effect of the coefficient B_1 is little larger for tall tanks ($L > 2R$). Dimensions of this tank are: radius $R = 0{\cdot}1$ m, shell height $L = 0{\cdot}6$ m, water depth $H = 0{\cdot}53$ m and shell thickness $h = 0{\cdot}001$ m. The material properties are the same of the previously studied shell. Numerical results for bulging modes are presented in Table 2 and were obtained by using the Donnell theory of shells.

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